NORTHERN BEACHES SECONDARY COLLEGE

MANLY SELECTIVE CAMPUS

HIGHER SCHOOL CERTIFICATE

Trial Examination

2017

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3hours
- Use **black** pen
- Write your Student Number at the top of each page
- Section I – Multiple Choice – use the Answer Sheet provided
- Section II – Free Response – use a separate booklet for **each** question.
- Board approved calculators and templates may be used.
- Reference sheet provided.

Section I Multiple Choice

- 10 marks
- Attempt all questions

Section II – Free Response

- Questions 11-16 – 90 marks
- Each question is of equal value
- All necessary working should be shown in every question.

Weighting: 40%

<u>Section 1 Multiple Choice:</u> <u>Attempt Questions 1 – 10</u>

Answer questions on the provided answer sheet.
Allow approximately 15 minutes for this section.

Q. 1 36.1984 written in scientific notation, correct to 4 significant figures is:

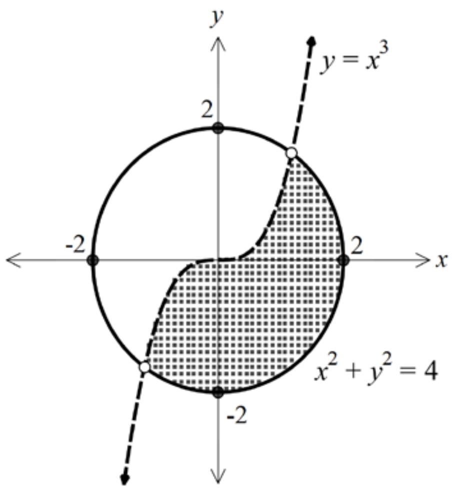
(A) 3.620×10

(B) 3.62×10

(C) 3.620×10^{-1}

(D) 3.6198×10^{-1}

Q. 2 The inequalities which define the shaded region shown in the diagram are:



(A) $x^2 + y^2 \geq 4$ and $y < x^3$

(B) $x^2 + y^2 \geq 4$ and $y \geq x^3$

(C) $x^2 + y^2 \leq 4$ and $y > x^3$

(D) **$x^2 + y^2 \leq 4$ and $y < x^3$**

Q.3 A line in the form of $px + qy = 5$ passes through the points (0, 5) and (2, 1). The gradient (m) and y-intercept (b) are:

(A) $m = 1$ $b = 2$

(B) $m = -2$ $b = 5$

(C) $m = 5$ $b = -2$

(D) $m = 2$ $b = 1$

Q. 4 The fourth, fifth and sixth terms of an arithmetic series are –3, 5 and 13 respectively. The first term of this series is:

(A) 21

(B) –11

(C) –27

(D) 8

Q. 5 Given $\log\frac{a}{b} + \log\frac{b}{a} = \log(a+b)$, then the correct statement below is:

(A) $a + b = 1$

(B) $a - b = 1$

(C) $a = b$

(D) $a^2 - b^2 = 1$

Q. 6 The function $f(x)$ is defined by $f(x) = \begin{cases} \dfrac{4}{x} & : x > 1 \\ 4^x & : x \leq 1 \end{cases}$.

What is the value of $f(0.5) + f(2)$?

(A) 4

(B) 10

(C) 18

(D) 24

Q. 7 $\displaystyle\int \frac{x-4}{x^2}\,dx$ can be expressed as:

(A) $\frac{1}{2}\ln x^2 + 4x + C$

(B) $\ln x^2 + \frac{4}{x} + C$

(C) $\ln x - 4x + C$

(D) $\ln x + \frac{4}{x} + C$

Q. 8 Which of the following statements is true if $(2,-5)$ is a minimum turning point of $f(x)$ and $f(x) = -f(-x)$?

(A) $(-2,-5)$ is a maximum turning point of $f(x)$

(B) $(-2,5)$ is a minimum turning point of $f(x)$

(C) $(-2,-5)$ is a minimum turning point of $f(x)$

(D) $(-2,5)$ is a maximum turning point of $f(x)$

Q. 9 The quadratic equation $x^2 - 9x + 16 = 0$ has roots α and β.
The value of $\sqrt{\alpha} + \sqrt{\beta}$ is:

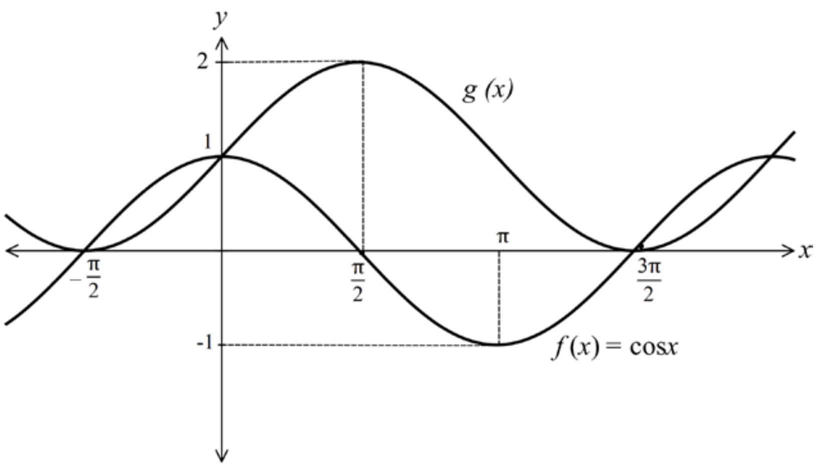
(A) 3

(B) $\sqrt{13}$

(C) $\sqrt{17}$

(D) 17

Q. 10 The diagram shows the graphs of $f(x) = \cos x$ and $g(x)$.



The equation of $g(x)$ is:

(A) $g(x) = f\left(x - \frac{\pi}{2}\right) + 2$

(B) $g(x) = f\left(x + \frac{\pi}{2}\right) + 1$

(C) $g(x) = f\left(x - \frac{\pi}{2}\right) + 1$

(D) $g(x) = f\left(x + \frac{\pi}{2}\right) + 2$

End of Multiple Choice

Section II 90 marks

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations

Question 11: Start A New Booklet 15 Marks

a. Fully factorise $16-2x^3$. **2**

b. Solve $|2x-5|<1$. **2**

c. Evaluate $\lim\limits_{x\to -1} \dfrac{2x^2+3x+1}{x+1}$ **2**

d. Find $\displaystyle\int_0^{\frac{\pi}{3}} \cos\left(\frac{x}{2}\right)\, dx$ **2**

e. Find the values of *p* and *q* for which $p+q\sqrt{6} = \dfrac{12\sqrt{6}}{\sqrt{6}-2}$. **2**

f. Sketch the graph of $(x-2)^2+y^2=4$. **2**

g. Show that $\dfrac{d}{dx}\left[2x(x-4)^3\right]=8(x-1)(x-4)^2$. **3**

End of Question 11

Question 12 Start a New Booklet 15 Marks

a. The vertices of ΔABC are A(1, 8) B(7, 4) and C(1,0) as shown below, where AQ is the altitude.

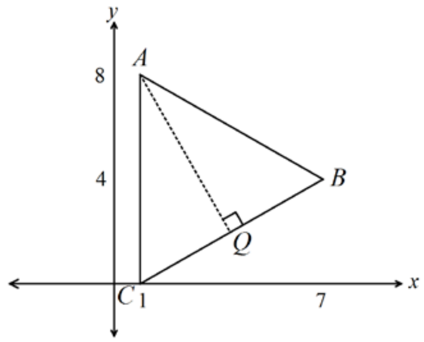


Diagram not to scale

i. Show the gradient of AQ is $-\frac{3}{2}$ **1**

ii. Hence, determine the equation of the altitude AQ in general form. **1**

iii Hence determine the length of BQ. (Leave your answer in exact form) **2**

b. In a geometric series, the fifth term is $\frac{1}{9}$ and the eight term is $\frac{1}{243}$.
Determine the value of the common ratio (r). **2**

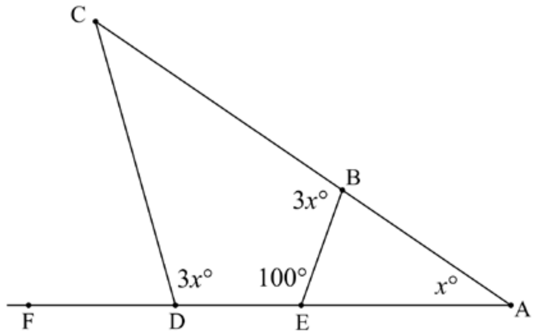
c. What is the value of $\sum_{m=1}^{\infty} 5\left(\frac{2}{5}\right)^{m-1}$? **2**

d. Find $\int \frac{1}{(2x+3)^5}\, dx$. **2**

Question 12 continues on the next page.

Question 12 continued:

e.

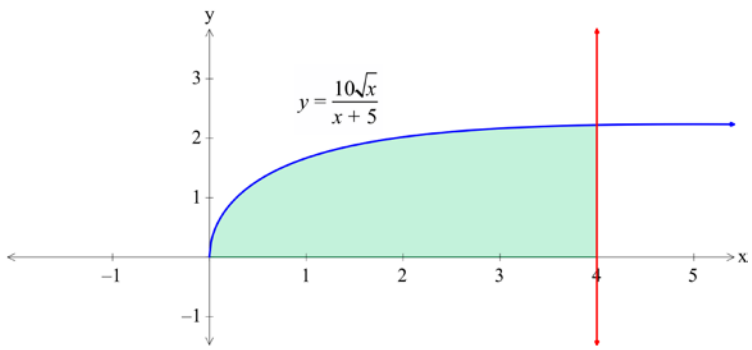


The points B and E lie on the sides AC and AD respectively of Δ ACD.
The point F lies on AD produced, as shown in the diagram.

(i) Copy the diagram into your answer booklet.
Make your diagram one third of a page.

(ii) Find the value of x, giving reasons. **2**

f The area bounded by the curve $f(x) = \dfrac{10\sqrt{x}}{x+5}$, the x- axis and the line $x = 4$ is shown in the diagram below.



i. Using the Trapezoidal Rule with 5 functional values, determine the approximate area of the shaded region.
Give your answer correct to two decimal places. **2**

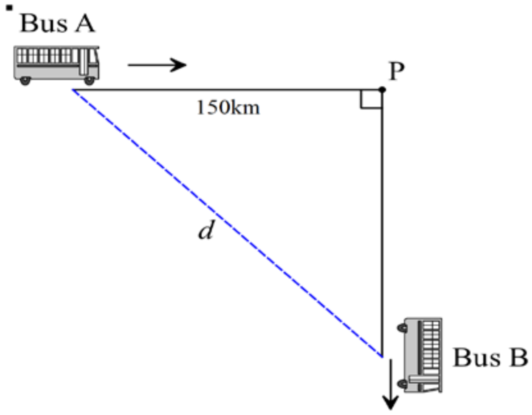
ii. Is the estimate calculated in part i) greater or less than the exact area ?
Give a reason to justify your answer. **1**

End of Question 12

Question 13 Start A New Booklet 15 Marks

a. Find . $\int (1 + \tan x)\ dx$. **2**

b. Two school buses are travelling along straight roads which intersect at right angles at the point P, as shown in the diagram.



Initially, Bus A is 150km due west of P and is travelling towards P at 50km/hr. At the same time Bus B leaves P and travels due south at 40km/hr. Let d km be the distance between Bus A and Bus B at t hours after the buses start moving.

i. Show that Bus A is $(150 - 50t)$ km from P after t hours **1**

ii. Show that **2**

$$d = \sqrt{4100t^2 - 15000t + 22500}\ ,$$

where d is the distance between the buses when $0 < t < 3$.

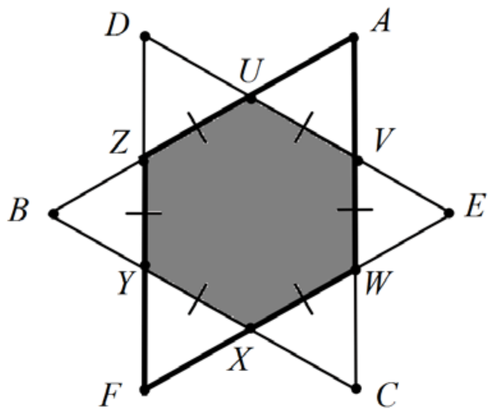
ii. Find the value of t which gives the minimum value of d. **2**
Answer correct to one decimal place.

Question 13 continues on the next page.

Question 13 continued:

c. Find the value of the derivative of $y = 4\tan(2x) - \frac{4x^2}{\pi}$ when $x = \frac{\pi}{12}$ **2**

d. The diagram shows a six pointed star which is drawn using two triangles, ΔABC and ΔFDE. The intersection of the two triangles is a regular hexagon.



i. Show that ΔAVU is an equilateral triangle. **2**

ii. Similarly ΔVEW, ΔWCX, ΔXFY, ΔYBZ and ΔZDU are all equilateral triangles. **2**

Prove that $ZAWF$ is a rhombus.

e. Find, in simplest form, $\frac{d}{dx}\left(\frac{\cos x}{1 - \sin x}\right)$. **2**

End of Question 13

Question 14 Start A New Booklet **15 Marks**

a. Sketch the graph of $y = \ln(x-4)$, clearly indicating the *x*-intercept and any asymptotes. **2**

b. Consider the curve with equation $y = 4x^4 + 8x^3 + 1$.

i. Find the stationary points and determine their nature. **4**

ii. Show that the point (-1, -3) is a point of inflexion. **2**

iii. Sketch the curve, labelling the stationary points, points of inflexion and *y*-intercept. **3**

c. A particle moves in a straight line such that its displacement *x* in metres is given by

$x = 70e^{-\frac{t}{10}} - 20t$, where *t* is time in seconds.

i. Find the initial displacement of the particle. **1**

ii. Will the particle ever come to rest? **2**
Justify your answer using appropriate calculations.

iii. Find the distance travelled by the particle in the first 3 seconds.

Give the answer correct to 2 decimal places. **1**

End of Question 14

Question 15 Start A New Booklet 15 Marks

a. The curve *f(x)* has a minimum turning point at (2, -10). The second derivative is given by the equation $f''(x) = 12x - 10$. **3**
Determine the equation of *f(x)*.

b. Determine the value of *k* given $\int_{-2}^{2} (x^7 - x + k)\ dx = 16$ where *k* is constant. **2**

c. The amount of caffeine, $C(t)$, in milligrams in your system after drinking a cappuccino is given by

$$C(t) = 105e^{-kt},$$

where *k* is a constant and *t* is the time in hours that have passed since drinking the cappuccino.

(i) After one hour the caffeine in your system has decreased by 40%. Find the exact value of *k*. **2**

(ii) When will there be 10 milligrams of caffeine remaining in your system? Give the answer correct to 2 significant figures. **1**

d. A quadratic function is defined by $f(x) = x^2 - 2kx + (2k + 3)$.

i) Find the values of *k* for which the equation $f(x) = 0$ has two real roots. **2**

ii) Find the values of *k* for which the solutions to $f(x) = 0$ are both positive. **2**

d. The rate at which the height of a Jacaranda tree grows is given by

$$\frac{dh}{dt} = \frac{110}{(t+4)^2} \text{ metres per year},$$

where *h* is the height of the tree in metres and *t* is the number of years that have passed since the tree was an established seedling with a height 0.5 m.

Find the height of the tree when $t = 5$, correct to 1 decimal place. **3**

End of Question 15

Question 16 Start A New Booklet **15 Marks**

a. Penny borrows \$250 000 to be repaid at a reducible interest rate of 0.4% per month. Let \$$A_n$ be the amount owing at the end of n months and \$$M$ be the monthly repayment.

(i) Show that $A_2 = 250\ 000(1.004)^2 - M(1 + 1.004)$ **2**

(ii) Show that $A_n = 250\ 000(1.004)^n - M\left(\dfrac{(1.004)^n - 1}{0.004}\right)$ **1**

(iii) If she repays the amount in 120 months, then show that M = \$2627, to the nearest dollar. 1

(iv) Penny decides to pay off the loan by making monthly payments of \$3500 instead of \$2627. **3**

Show that Penny will make 84 repayments of \$3500 and then a final part payment of \$1001.64.

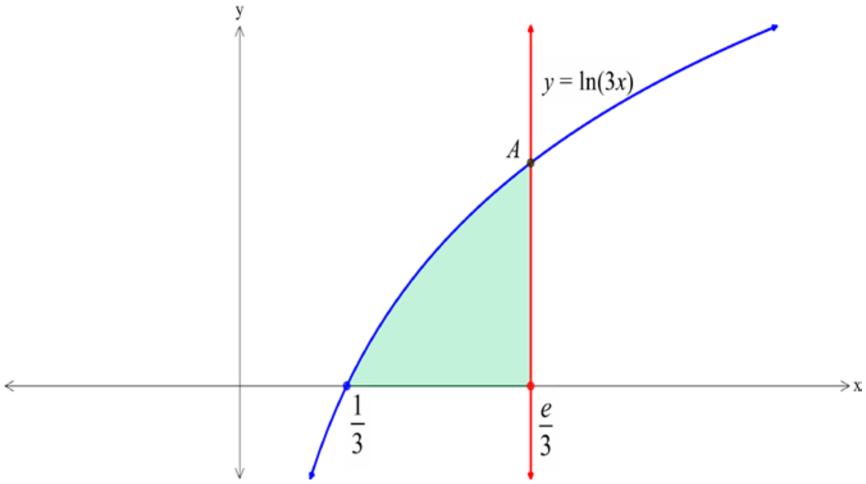
b. i Show $\dfrac{\sec\theta}{\operatorname{cosec}\theta} = \tan\theta$. **1**

ii Hence, or otherwise, solve $3\sec^2\left(\dfrac{x}{2}\right) = \operatorname{cosec}^2\left(\dfrac{x}{2}\right)$ for $0 \le x \le 2\pi$. **3**

Question 16 continues on the next page.

Question 16 continued:

b. The shaded region shown below is rotated around the y-axis.
The point A is the intersection of the curve $y = \ln(3x)$ and the line $x = \dfrac{e}{3}$.



i Determine the y – value of point A. **1**

ii Determine the exact volume of the solid of revolution formed. **3**

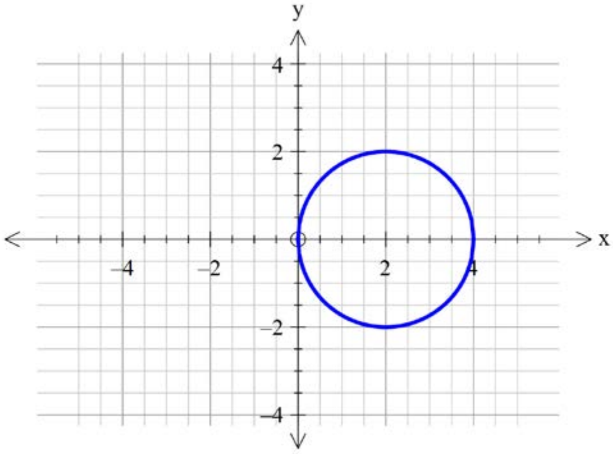
<u>End of Examination</u>

MSC HSC Trial Examination 2017- Solutions

Q1	A	$3.61984 \times 10^1 = 3.620 \times 10^1$ (4 sig)
Q2	D	Inside the circle and below but not including the cubic.
Q3	B	sub (0 , 5) into $px + qy = 5$ $p(0) + q(5) = 5$ $5q = 5$ $q = 1$ $\therefore \quad px + y = 5$ sub (2, 1) into $px + y = 5$ $p(2) + 1 = 5$ $p = 2$ $\therefore \quad 2x + y = 5$ $m = {}^-2$ $b = 5$
Q4	C	$T_4 = {}^-3$ $T_5 = 5$ $T_6 = 13$ $\therefore \; d = 8$ $T_1 = T_4 - 3d$ $= {}^-3 - 3(8)$ $= {}^-27$
Q5	A	$\log\left(\frac{a}{b} * \frac{b}{a}\right) = \log(a + b)$ $a + b = 1$
Q6	A	$f(0.5) = 4^{\frac{1}{2}} = 2 \quad f(2) = \frac{4}{2} = 2$ $f(0.5) + f(2) = 4$
Q7	D	$\int \frac{x}{x^2} - \frac{4}{x^2}\,dx = \int \frac{1}{x} - 4x^{-2}\,dx$ $= \ln x - \frac{4x^{-1}}{-1} + C$ $= \ln x + \frac{4}{x} + C$
Q8	D	$f(x) = -f(-x) \Rightarrow f(-x) = -f(x)$ $\therefore$ function is odd If minimum turning point at $(2 , -5)$ then maximum turning point (-2 , 5)
Q9	C	

		$\alpha + \beta = 9$ $\alpha \times \beta = 16$ $(\sqrt{\alpha} + \sqrt{\beta})^2 = (\sqrt{\alpha})^2 + 2\sqrt{\alpha}\sqrt{\beta} + (\sqrt{\beta})^2$ $= \alpha + \beta + 2\sqrt{\alpha\beta}$ $= 9 + 2\sqrt{16}$ $= 9 + 8$ $= 17$ $\therefore \sqrt{\alpha} + \sqrt{\beta} = \sqrt{17}$
Q10	C	Curve has been lifted by 1 unit vertically and translated $\frac{\pi}{2}$ units to the right therefore $g(x) = f\left(x - \frac{\pi}{2}\right) + 1$

Q11a	$16 - 2x^3 = 2(8 - x^3)$ $= 2(2 - x)(4 + 2x + x^2)$	2 marks for correct solution from correct working 1 mark for taking out common factor
Q11b	$-1 < 2x - 5 < 1$ $4 < 2x < 6$ $\therefore\ 2 < x < 3$	2 marks for correct solution from correct working
Q11c	$\lim_{x \to -1} \frac{2x^2 + 3x + 1}{x + 1} = \lim_{x \to -1} \frac{(2x + 1)(x + 1)}{x + 1}$ $= \lim_{x \to -1} 2x + 1$ $= -2 + 1$ $= -1$	2 marks for correct solution from correct working
Q11d	$\int_0^{\frac{\pi}{3}} \cos\left(\frac{x}{2}\right)\ dx$ $= 2\left[\sin\frac{x}{2}\right]_0^{\frac{\pi}{3}}$ $= 2\left(\sin\frac{\pi}{6} - \sin 0\right)$ $= 2 \times \frac{1}{2} = 1$	2 marks for correct solution from correct working
Q11e	$\frac{12\sqrt{6}}{\sqrt{6} - 2} = \frac{12\sqrt{6}}{\sqrt{6} - 2} \times \frac{\sqrt{6} + 2}{\sqrt{6} + 2}$ $= \frac{72 + 24\sqrt{6}}{6 - 4}$ $= \frac{72 + 24\sqrt{6}}{2}$ $= 36 + 12\sqrt{6}$ $\therefore\ p = 36$ and $q = 12$	2 marks for correct solution from correct working

Q11f	 Centre (2,0) Radius=2	2 marks for correct graph
Q11g	$U = 2x \qquad V = (x-4)^3$ $U' = 2 \qquad V' = 3(x-4)^2$ $\frac{d}{dx}[2x(x-4)^3] = VU' + UV'$ $= 2(x-4)^3 + 6x(x-4)^2$ $= 2(x-4)^2[(x-4)+3x]$ $= 2(x-4)^2(4x-4)$ $= 8(x-4)^2(x-1)$	3 marks for correct solution from correct working

Markers Comments

Q12a(i)	$m_{AQ} \times m_{CB} = -1$ $mAQ = -\frac{1}{m_{CB}}$ $m_{CB} = \frac{4-0}{7-1} = \frac{2}{3}$ $\therefore \quad m_{AQ} = -\frac{3}{2}$	1 mark – correct demonstration from gradient of CB
Q12a(ii)	$y = mx + b$ $y = -\frac{3}{2}x + b$ $x = 1 \Rightarrow y = 8$ $8 = -\frac{3}{2} + b$ $b = 8 + \frac{3}{2} = \frac{19}{2}$ $y = -\frac{3}{2}x + 9.5$ $2y = -3x + 19$ $3x + 2y - 19 = 0$	1 mark – correct formula ie. Both gradient and y-intercept correct
a-iii	$D = \frac{\|ax + by + c\|}{\sqrt{a^2 + b^2}}$ $(x,y) \Rightarrow (7,4)$ $3x + 2y - 19 = 0$ $D = \frac{\|3 \times 7 + 2 \times 4 - 19\|}{\sqrt{9+4}}$ $= \frac{\|10\|}{\sqrt{13}}$ $= \frac{10\sqrt{13}}{13}$	2 marks – correct solution 1 mark – correct substitution into formula

b	$T_n = ar^{n-1}$ $\dfrac{T_8}{T_5} = \dfrac{ar^7}{ar^4} = r^3 = \left(\dfrac{1}{243}\right) \div \dfrac{1}{9}$ $r^3 = \dfrac{1}{27}$ $r = \dfrac{1}{3}$	2 marks – correct solution. 1 mark – determining expression for r^3
c	$\sum_{m=1}^{\infty} 5\left(\dfrac{2}{5}\right)^{m-1}$ - expression is limiting sum therefore $T_1 = a = 5\left(\dfrac{2}{5}\right)^0 = 5$ $r = \dfrac{2}{5}$ $S_\infty = \dfrac{a}{1-r}$ $= \dfrac{5}{1-\dfrac{2}{5}}$ $= \dfrac{5}{\dfrac{3}{5}}$ $= \dfrac{25}{3}$	2 marks – correct solution 1 mark – correct first term and common ratio identified. -
d	$\int \dfrac{1}{(2x+3)^5}\, dx$ $= \int (2x+3)^{-5}\, dx$ $= \dfrac{(2x+3)^{-4}}{2 \times -4} + C$ $= -\dfrac{1}{8(2x+3)^4} + C$	2marks – correct solution 1 mark – incorrect denominator

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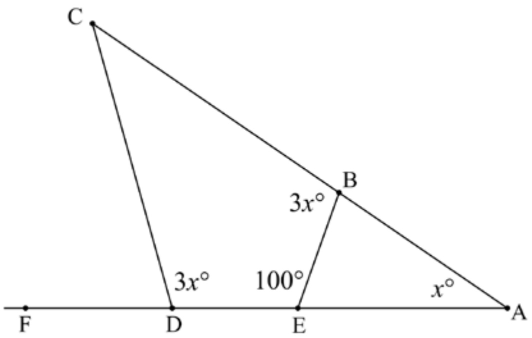

$\angle BEA = 180 - 100 = 80$ straight angle
$\angle CBE = \angle BEA + \angle BAE$ ext ∠ of Δ = Σ opp interior ∠
$3x = 80 + x$
$2x = 80$
$x = 40$
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<td>2 marks – correct answer with reasons

1 mark – correct answer with incomplete reasons</td>
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<td>f-i</td>
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<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>$y = \dfrac{10\sqrt{x}}{x+5}$</td><td>0</td><td>$\dfrac{10}{6} = \dfrac{5}{3}$</td><td>$\dfrac{10\sqrt{2}}{7}$</td><td>$\dfrac{10\sqrt{3}}{8} = \dfrac{5\sqrt{3}}{4}$</td><td>$\dfrac{20}{9}$</td></tr>
</table>

$$A \cong \frac{h}{2}\{ y_0 + 2(y_1 + y_2 + y) + y_4 \}$$
$$\cong \frac{1}{2} \times \left(0 + 2\left(\frac{5}{3} + \frac{10\sqrt{2}}{7} + \frac{5\sqrt{3}}{4}\right) + \frac{20}{9}\right)$$
$$\cong 6.96$$
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<td>2 marks – correct solution

1 mark – correct table or similar</td>
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<td>f-ii</td>
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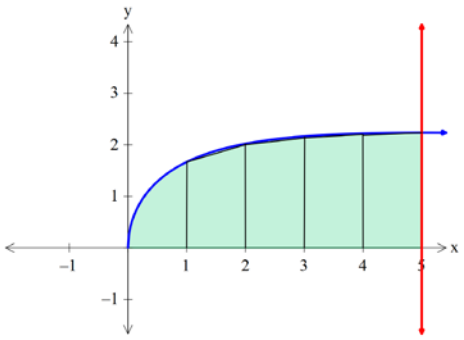

As the curve is concave down, the trapeziums would lie under the curve hence estimate is an underestimate.
</td>
<td>1 mark – correct answer with reasons.</td>
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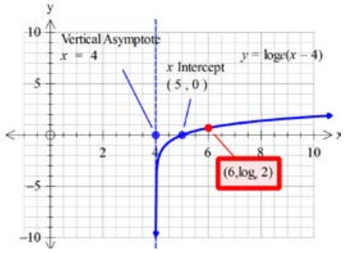
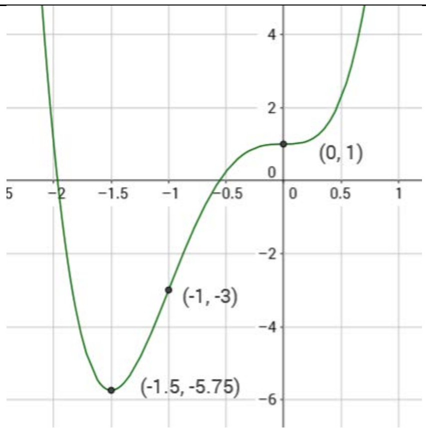
Markers Comments.

12-a-i a number of students incorrectly interpreted as midpoint

Q13a	$\int (1+\tan x)dx = \int \left(1+\frac{\sin x}{\cos x}\right)dx$ $= x - \ln \cos x + c$	2 marks correct solution from correct working 1 mark correct $\tan x = \frac{\sin x}{\cos x}$
Q13b-i	Distance from P = 150 - distance travelled = 150 – 50t	1 mark – correct demonstration.
Q13b-ii	$d = \sqrt{(150-50t)^2 + (40x)^2}$ $= \sqrt{22500 - 2 \times 150 \times 50t + 2500t^2 + 1600t^2}$ $= \sqrt{4100t^2 - 15000t + 22500}$	2 marks correct solution from correct working 1 mark correct substitution into Pythagoras' Theorem
13b-iii	As $4100t^2 - 15000t + 22500$ is a parabola with a = 4100, the parabola is concave up and therefore a minimum. $f'(t) = 8200t - 15000$ stationary point at $f'(t) = 0$ $0 = 8200t - 15000$ $t = \frac{15000}{8200}$ $= 1{\cdot}82926.....$ $= 1{\cdot}8$	2 marks correct solution from correct working. 1 marks correct x with no justification for minimum value
13c	$y = 4\tan(2x) - \frac{4x^2}{\pi}$ $y' = 4 \times \sec^2(2x) \times 2 - \frac{8x}{\pi}$ $= 8\left(\sec^2(2x) - \frac{x}{\pi}\right)$ at $x = \frac{\pi}{12}$ $y' = 8\left(\sec^2\left(\frac{2\pi}{12}\right) - \frac{\frac{\pi}{12}}{\pi}\right)$ $= 8\left(\sec^2\left(\frac{\pi}{6}\right) - \frac{1}{12}\right)$ $= 8\left(\left(\frac{2}{\sqrt{3}}\right)^2 - \frac{1}{12}\right)$ $= 8\left(\frac{4}{3} - \frac{1}{12}\right)$ $= 10$	2 marks correct solution from correct working. 1 mark correct derivative.

13di	ZUVWXY regular hexagon (given) Interior angle = $\dfrac{180(6-2)}{6}$ $= 120°$ $\angle AUV = \angle AVU$ $= 180° - 120°$ (straight $\angle$) $= 60°$ $\angle VAU = 180° - (60° + 60°)$ ($\angle$ sum of Δ) $= 60°$ $\therefore$ $\Delta\, AVU$ is equilateral	2 marks correct solution from correct working with reasons. 1 mark finding an interior angle of hexagon
13dii	$\angle ZAW = \angle ZFW$ $= 60°$ (equilateral $\Delta\, s$) $\angle AZF = \angle AWF$ $= 120°$ ($\angle$ of regular hexagon) $\therefore ZAWF$ is aparallelogram (opposite $\angle$ =) AU = AV ($\Delta\, AVU$ equilateral) ZU = VW (given) $\therefore$ ZA = WA $\therefore ZAWF$ is a rhombus (parallelogram with a pair of adjacent sides =)	2 marks correct solution from correct working with reasons. 1 mark proving ZAWF is a parallelogram
13e	$\dfrac{d}{dx}\left(\dfrac{\cos x}{1-\sin x}\right)$ $u = \cos x \quad v = 1 - \sin x$ $u' = -\sin x \quad v' = -\cos x$ $\dfrac{d}{dx}\left(\dfrac{\cos x}{1-\sin x}\right) = \dfrac{-\sin x(1-\sin x) - \cos x(-\cos x)}{(1-\sin x)^2}$ $= \dfrac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}$ $= \dfrac{1-\sin x}{(1-\sin x)^2}$ $= \dfrac{1}{1-\sin x}$	2 marks correct solution from correct working 1 mark correct u' and v'

<table>
<tr>
<td>Q14a)</td>
<td>
</td>
<td>2 marks correct answer with 2nd pt

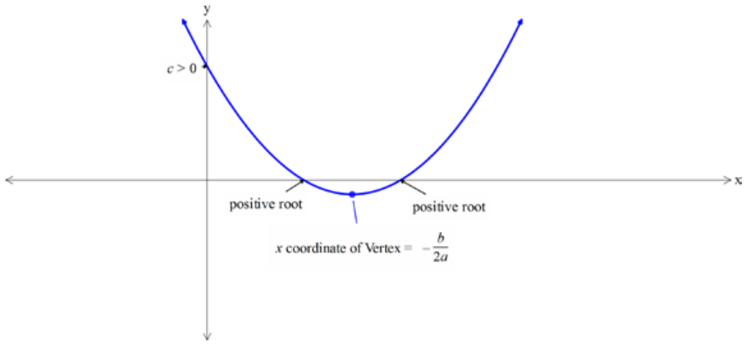
1 mark correct asymptote and x-int</td>
</tr>
<tr>
<td>bi)</td>
<td>$y' = 16x^3 + 24x^2$
$y' = 0$ for a $S.P.$
$8x^2(2x + 3) = 0$
$x = 0$ and $x = -\frac{3}{2}$
$y = 1$ and $y = -\frac{23}{4}$ respectively
Using a slope table
<table>
<tr><td>x</td><td>-2</td><td>-1.5</td><td>-1</td><td>0</td><td>1</td></tr>
<tr><td>y’</td><td>-32</td><td>0</td><td>8</td><td>0</td><td>40</td></tr>
<tr><td>slope</td><td>\</td><td>-</td><td>/</td><td>-</td><td>/</td></tr>
</table>
$\left(-\frac{3}{2}, -\frac{23}{4}\right)$ is a minimum and "(0,1) a horizontal point of inflexion
Or use of y'' to show concavity change
$y'' = 0$ is not sufficient for a point of inflection</td>
<td>4 marks: correct answer
3 marks: correct coordinates and their nature, showing HPOI
2 marks: correct co-ords
1 mark: correct first derivative</td>
</tr>
<tr>
<td>ii)</td>
<td>$y'' = 48x^2 + 48x$
$y'' = 0$ for possible points of inflexion
$48x(x + 1) = 0$
$x = -1, 0$
or sub x=1 into 2nd derivative
this point of inflection is NOT a horizontal point of inflection</td>
<td>2 marks: correct answer
1 mark: correct second derivative</td>
</tr>
<tr>
<td>iii)</td>
<td>
</td>
<td>3 marks: correct answer
2 marks: correct shape
1 mark correct stationary points

Cfe applicable

NOTE some very poor graphs were presented</td>
</tr>
</table>

c)i	$x = 70e^{0} - 20(0)$ $x = 70\text{m}$	1 mark correct answer
ii)	$= -7e^{\frac{t}{10}} - 20$ $= 0$ for particle at rest $e^{-\frac{t}{10}} \neq -\frac{20}{7}$ since $e^{-\frac{t}{10}} > 0$ for all	
iii)	from i) t=0 and x=70 t=3 x=-8.14 therefore distance travelled is 78.14m	1 mark correct answer

Q15 a	$f''(x) = 12x - 10$ $f'(x) = 6x^2 - 10x + c_1$ stationary points at $f'(x) = 0$ when $x = 2$ $f'(2) = 6(2)^2 - 10(2) + c_1 = 0 \Rightarrow c_1 = -4$ $\therefore f'(x) = 6x^2 - 10x - 4$ $f(x) = 2x^3 - 5x^2 - 4x + c_2$ sub point $(2, -10)$ $-10 = 2(2)^3 - 5(2)^2 - 4(2) + c_2 \Rightarrow c_2 = 2$ $\therefore f(x) = 2x^3 - 5x^2 - 4x + 2$	3 marks correct solution from correct working 2 marks- partial correct/ one error 1 mark- partial correct with one required process
Q15 b	Method 1 $\int_{-2}^{2} x^7 - x + k\, dx = \left[\frac{x^8}{8} - \frac{x^2}{2} + kx\right]_{-2}^{2}$ $= \left(\frac{256}{8} - \frac{4}{2} + 2k\right) - \left(\frac{256}{8} - \frac{4}{2} - 2k\right)$ $= 4k$ $\therefore \quad 4k = 16$ $k = 4$ Method 2 $\int_{-2}^{2} (x^7 - x + k)\ dx$ $= \int_{-2}^{2} (x^7 - x)\ dx + \int_{-2}^{2} k\ dx$ $= 0 + \left[kx\right]_{-2}^{2}$ (*as* first function is an odd function) $\therefore 2k - (-2k) = 16$ $4k = 16$ $k = 4$	2 marks correct solution from correct working 1 mark for integration
Q15 c(i)	when $t = 0$ $\quad C(0) = 105$ when $t = 1$ $\quad C(1) = 0.6 \times 105 = 63$ and $C(1) = 105e^{-k}$ $\therefore 105e^{-k} = 63$ $e^{-k} = \frac{63}{105}$ $-k = \ln\left(\frac{63}{105}\right)$ $k = -\ln(0.6)$	2 marks correct solution from correct working 1 mark for using 0.4 in equation and solving with correct process Common error: Using 40% instead of 60%

Q15 c(ii)	$105e^{-kt} = 10$ $e^{-kt} = \frac{10}{105} = \frac{2}{21}$ $-kt = \ln\left(\frac{2}{21}\right)$ $t = \ln\left(\frac{2}{21}\right) \div \ln(0.6)$ $t \approx 4.6$ hours (2 sigfig)	1 marks correct solution from correct working 1 mark if error carried from part i) showing correct process Common error: Rounding to 3 sig fig (4.60)
Q15 d(i)	For 2 real roots $\Delta > 0$ $\Delta = b^2 - 4ac$ $= (-2k)^2 - 4(2k+3)$ $= 4k^2 - 8k - 12$ $4k^2 - 8k - 12 > 0$ $k^2 - 2k - 3 > 0$ $(k+1)(k-3) > 0$ $\therefore \quad k < -1$ or $k > 3$	2 marks correct solution from correct working 1 mark for discriminant
Q15 d(ii)	Method 1 For there to be two roots, *k* must satisfy $k < -1$ or $k > 3$ Since a>0, parabola is concave up, for there to be two positive roots *k* must satisfy additional conditions of constant term > 0 and $-\frac{b}{2a} > 0$ - see diagram below.  $\therefore c > 0$ i.e. $2k + 3 > 0$ $k > -\frac{3}{2}$ Also, axis of symmetry $x = -\frac{b}{2a}$ must be > 0 $x = -\frac{b}{2a} = \frac{2k}{2(1)} = k$ $\therefore k > 0$ Therefore to satisfy all conditions, *k* must simultaneously satisfy	2 marks correct solution from correct working 1 mark for considering one condition Common error: Not finding the other condition of k>0 Using the quadratic equation and then solving for x>0 resulted in algebraic error (If no error, successfully found k>-3/2 condition but failed to find k>0 condition)

	$k > -\frac{3}{2}$, $k > 0$ and $k > 3$ therefore $k > 3$ satisfies all conditions. Method 2 $\alpha + \beta > 0$ and $\alpha\beta > 0$ $\alpha + \beta = -\frac{b}{a} = 2k$ $\therefore \quad 2k > 0 \Rightarrow k > 0$ $\alpha\beta = \frac{c}{a} = 2k + 3$ $\therefore 2k + 3 > 0 \Rightarrow k > -\frac{3}{2}$ Considering all conditions $k > 3$	
Q15 e	Method 1 $h = \int_0^5 110(t+4)^{-2}dt + 0.5$ $= 110\left[\frac{(t+4)^{-1}}{-1}\right]_0^5 + 0.5$ $= 110\left[-\frac{1}{t+4}\right]_0^5 + 0.5$ $= 110\left[-\frac{1}{9} + \frac{1}{4}\right] + 0.5$ $\approx 15.8m$ Method 2 $h = \int 110(t+4)^{-2}dt$ $= -\frac{110}{t+4} + c$ when $t = 0$, h=0.5 $0.5 = -\frac{110}{4} + c$ $c = 28$ $\therefore \quad h = -\frac{110}{t+4} + 28$ *when* $t = 5$ $h = -\frac{110}{9} + 28 \approx 15.8$	3 marks correct solution from correct working 2 marks for definite integral but omitting the 0.5 1 mark for integration 3 marks correct solution from correct working 2 marks for integration and constant 1 mark for integration

Question 16		
16 a i)	P = $ 250 000, R = 1.004 $A_n = A_{n-1}R - M$ $\Rightarrow A_1 = 250\ 000(1{\cdot}004) - M$ $A_2 = A_1 R - M$ $\Rightarrow A_2 = [250\ 000(1{\cdot}004) - M](1{\cdot}004) - M$ $A_2 = 250\ 000(1{\cdot}004)^2 - 1{\cdot}004M - M$ $A_2 = 250\ 000(1{\cdot}004)^2 - M(1 + 1{\cdot}004)$	2 Marks: Correct solution 1 Mark: correct expression for A_1
16 a ii)	$A_n = 250\ 000(1{\cdot}004)^n - M(1 + 1{\cdot}004 + .. + 1{\cdot}004^{n-1}$ $S_n = a\left[\dfrac{R^n - 1}{R - 1}\right]$ and $a = 1$, $R = 1{\cdot}004$ $= 250\ 000(1{\cdot}004)^n - M\left[\dfrac{1{\cdot}004^n - 1}{0{\cdot}004}\right]$	1 Mark: Correct solution Must show serries up to and including 1.004^{n-1}
16 a iii)	$A_{120} = 250\ 000(1{\cdot}004)^{120} - M\left[\dfrac{1{\cdot}004^{120} - 1}{0{\cdot}004}\right]$ $0 = 250\ 000(1{\cdot}004)^{120} - M(153{\cdot}631...)$ $M = \dfrac{250\ 000(1{\cdot}004)^{120}}{153{\cdot}631....}$ $M = 2627{\cdot}2655... \approx \$2627$	1 Mark: Correct solution
16 a iv)	$A_n = 250\ 000(1{\cdot}004)^n - 3500\left[\dfrac{1{\cdot}004^n - 1}{0{\cdot}004}\right]$ $0 = 250\ 000(1{\cdot}004)^n - 875\ 000(1{\cdot}004^n - 1)$ $0 = 250\ 000(1{\cdot}004)^n - 875\ 000(1{\cdot}004)^n + 875\ 000$ $0 = -625\ 000(1{\cdot}004)^n + 875\ 000$ $(1{\cdot}004)^n = \dfrac{875\ 000}{625\ 000}$ $n\log_e(1{\cdot}004) = \log_e\left(\dfrac{875}{625}\right)$ $n = \log_e\left(\dfrac{875}{625}\right) \div \log_e(1{\cdot}004) = 84{\cdot}286...$ $\therefore$ 84 full payments and part payment of $0{\cdot}286...(\$3500) = \$\ 1001{\cdot}64$	3 Marks: Correct solution. 2 Marks: Correct derivation of n = 84.286…. 1 Mark:
16 b i)	$\dfrac{\sec\theta}{\mathrm{cosec}\theta} = \left(\dfrac{1}{\cos\theta}\right) \div \left(\dfrac{1}{\sin\theta}\right) = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$	1 Mark: Correct solution

16 b ii)	$3\sec^2\left(\frac{x}{2}\right) = \text{cosec}^2\left(\frac{x}{2}\right) \Rightarrow \frac{3}{\cos^2\left(\frac{x}{2}\right)} = \frac{1}{\sin^2\left(\frac{x}{2}\right)}$ $\frac{\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)} = \frac{1}{3} \Rightarrow \tan^2\left(\frac{x}{2}\right) = \frac{1}{3}$ $\tan\left(\frac{x}{2}\right) = \pm\frac{1}{\sqrt{3}}$ $0 \leq x \leq 2\pi$ $\Rightarrow \quad 0 \leq \frac{x}{2} \leq \pi \Rightarrow \frac{x}{2} = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow x = \frac{\pi}{3}, \frac{5\pi}{3}$	
16 c i)	$x = \frac{e}{3} \rightarrow y = \log_e(3x) \Rightarrow y = \log_e\left(3.\frac{e}{3}\right) = \log_e(e) = 1$	
16 c ii)	$V_1 = \text{ cylinder} = \pi r^2 h = \pi\left(\frac{e}{3}\right)^2 \cdot 1 = \frac{\pi e^2}{9}$ $V_2 = \text{ solid of revolution} = \pi\int_0^1 x^2 dy$ $y = \log_e(3x) \rightarrow 3x = e^y \rightarrow x^2 = \left(\frac{e^y}{3}\right)^2 = \frac{e^{2y}}{9}$ $V_2 = \left(\frac{\pi}{9}\right)\int_0^1 e^{2y}\,dy = \frac{\pi}{9}\left[\frac{1}{2}e^{2y}\right]_0^1 = \frac{\pi}{18}[e^2 - 1]$ $V = V_1 - V_2 = \frac{\pi e^2}{9} - \frac{\pi}{18}[e^2 - 1]$ $V = \frac{\pi(e^2 + 1)}{18}$	